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BRIEF COMMUNICATIONS

A COMPARISON OF SOME VOID-FRACTION RELATIONSHIPS FOR CO-CURRENT GAS–LIQUID FLOW

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INTRODUCTION

The purpose of this short note is to show the similarity between six models and correlations which have been proposed for predicting void fractions in co-current gas–liquid flows. The correlations and models considered are:

(1) Homogeneous model;
(2) Zivi model (1963);
(3) Turner & Wallis two-cylinder model (1965);
(4) Lockhart & Martinelli correlation (1949);
(5) Thom correlation (1964);
(6) Baroczy correlation (1963).

HOMOGENEOUS MODEL

The homogeneous model for two-phase flow gives the following value of void fraction:

$$\alpha = \frac{x/\rho_G}{x/\rho_G + (1 - x)/\rho_L} \qquad [1]$$

where α is the void fraction, x is the quality, and ρ_G and ρ_L are the gas and liquid density respectively.

Equation [1] may be rewritten as

$$\frac{1-\alpha}{\alpha} = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right). \qquad [2]$$

ZIVI MODEL

Zivi (1963) derived a number of simple void-fraction models of which the simplest is as follows:

$$\alpha = \frac{1}{1 + \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{2/3}} \qquad [3]$$

or

$$\frac{1-\alpha}{\alpha} = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{2/3}. \qquad [4]$$

This model has proved particularly successful in predicting pressure drop (Bae *et al.* 1969) and heat transfer (Bae *et al.* 1969; Soliman *et al.* 1968) during condensation.

TURNER & WALLIS TWO-CYLINDER MODEL

Turner & Wallis (1965) have derived a simple model for void fraction which may be written as follows for "turbulent–turbulent" flow:

$$\frac{1-\alpha}{\alpha} = X^{0.8} \qquad [5]$$

where X is the Lockhart & Martinelli parameter given by

$$X = \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}\left(\frac{\mu_L}{\mu_G}\right)^{0.1} \qquad [6]$$

Combining [5] and [6] gives:

$$\frac{1-\alpha}{\alpha} = \left(\frac{1-x}{x}\right)^{0.72}\left(\frac{\rho_G}{\rho_L}\right)^{0.4}\left(\frac{\mu_L}{\mu_G}\right)^{0.08} \qquad [7]$$

where μ_L and μ_G are the liquid and gas viscosity, respectively.

It must be admitted, however, that this model is not a particularly good representation of experimental data.

LOCKHART & MARTINELLI CORRELATION

The Lockhart & Martinelli (1949) correlation is essentially of the following form

$$\frac{1-\alpha}{\alpha} = f(X). \qquad [8]$$

Figure 1 shows the Lockhart & Martinelli correlation plotted in the form $(1-\alpha)/\alpha$ versus X. It is evident from this figure that the graphical correlation is closely approximated by:

$$\frac{1-\alpha}{\alpha} = 0.28X^{0.71} \qquad [9]$$

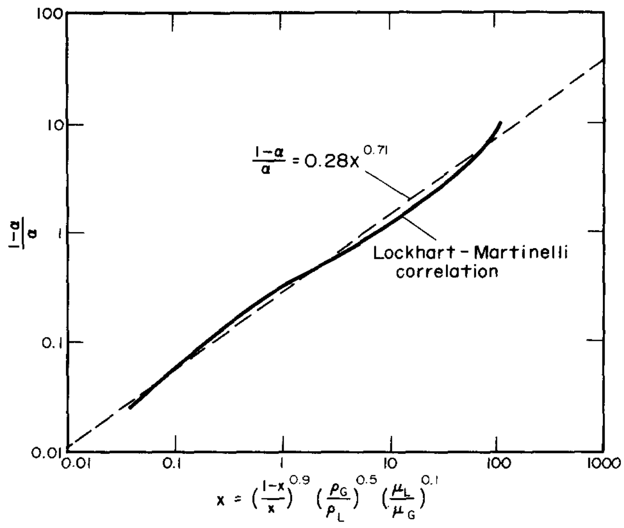

Figure 1. Transformed Lockhart & Martinelli correlation.

hence,

$$\frac{1-\alpha}{\alpha} = 0.28\left(\frac{1-x}{x}\right)^{0.64}\left(\frac{\rho_G}{\rho_L}\right)^{0.36}\left(\frac{\mu_L}{\mu_G}\right)^{0.07}. \qquad [10]$$

THOM CORRELATION

Thom (1964) correlated steam–water void fraction data by the following equation:

$$\alpha = \frac{\gamma x}{1 + x(\gamma - 1)} \qquad [11]$$

where γ is a "slip factor" which Thom expresses as a unique function of pressure. Equation [11] may be rewritten as

$$\frac{1-\alpha}{\alpha} = \frac{1}{\gamma}\left(\frac{1-x}{x}\right). \qquad [12]$$

Figure 2 shows γ plotted against a parameter Z, where $Z = (\rho_L/\rho_G)^{0.555}(\mu_G/\mu_L)^{0.111}$. It can be seen that Thom's parameter may be approximated by

$$\gamma = Z^{1.6} \qquad [13]$$

Combining [12] and [13] gives

$$\frac{1-\alpha}{\alpha} = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{0.89}\left(\frac{\mu_L}{\mu_G}\right)^{0.18}. \qquad [14]$$

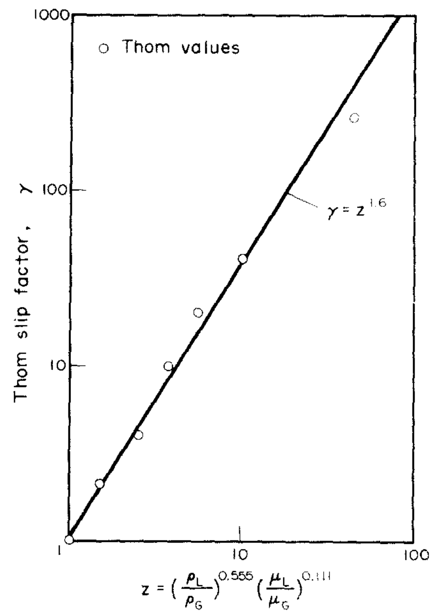

Figure 2. Transformed Thom correlation.

BAROCZY CORRELATION

Baroczy correlates void fraction data graphically in the form:

$$1 - \alpha = f_1(X, \Lambda) \quad [15]$$

where

$$\Lambda = \frac{\rho_G}{\rho_L}\left(\frac{\mu_L}{\mu_G}\right)^{0.2}. \quad [16]$$

Since X is given by $\{(1 - x)/x\}^{0.9}\Lambda^{0.5}$, [15] may be rewritten as

$$\frac{1-\alpha}{\alpha} = f_2\left(\frac{1-x}{x}, \Lambda\right). \quad [17]$$

The function f_2 is approximated over most of the range of x and Λ by

$$\frac{1-\alpha}{\alpha} = \left(\frac{1-x}{x}\right)^{0.74}\Lambda^{0.65} \quad [18]$$

This relationship is shown as the broken lines on figure 3. It can be seen that equation [18] is in agreement with Baroczy's curves for void fractions less than about 0.9. The region of discrepancy possibly corresponds to the annular flow regime.

Combining [16] and [18] gives

$$\frac{1-\alpha}{\alpha} = \left(\frac{1-x}{x}\right)^{0.74}\left(\frac{\rho_G}{\rho_L}\right)^{0.65}\left(\frac{\mu_L}{\mu_G}\right)^{0.13}. \quad [19]$$

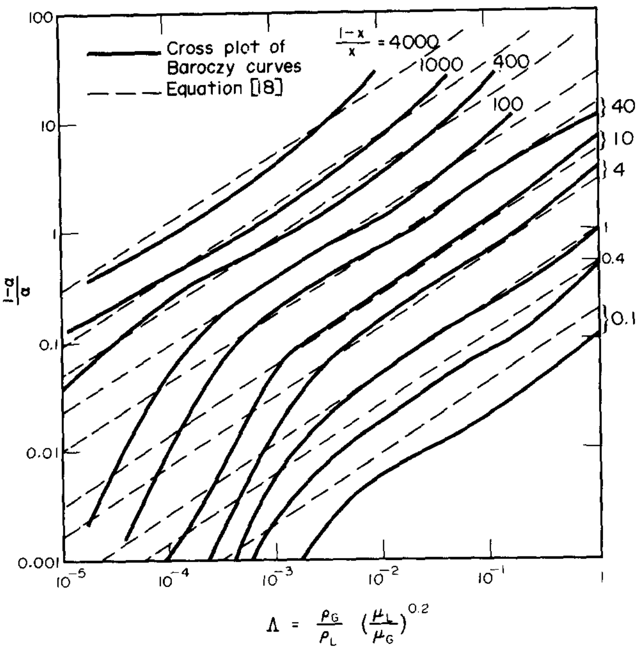

Figure 3. Transformed Baroczy correlation.

SUMMARY

It can be seen that the above models and correlations all take the following form:

$$\frac{1-\alpha}{\alpha} = A\left(\frac{1-x}{x}\right)^p\left(\frac{\rho_G}{\rho_L}\right)^q\left(\frac{\mu_L}{\mu_G}\right)^r \qquad [20]$$

where A is a constant. Explicitly in terms of the void fraction

$$\alpha = \frac{1}{1 + A\left(\frac{1-x}{x}\right)^p\left(\frac{\rho_G}{\rho_L}\right)^q\left(\frac{\mu_L}{\mu_G}\right)^r}. \qquad [21]$$

One is often interested in the slip ratio, U_G/U_L, which is given by:

$$\frac{U_G}{U_L} = \left(\frac{x}{1-x}\right)\left(\frac{\rho_L}{\rho_G}\right)\left(\frac{1-\alpha}{\alpha}\right) \qquad [22]$$

Table 1. Values of constants suggested by the various correlations and models

Correlation or model	A	p	q	r
Homogeneous model	1	1	1	0
Zivi model	1	1	0.67	0
Turner & Wallis model	1	0.72	0.40	0.08
Lockhart & Martinelli	0.28	0.64	0.36	0.07
Thom	1.	1	0.89	0.18
Baroczy	1	0.74	0.65	0.13

combining [20] and [22] gives

$$\frac{U_G}{U_L} = A\left(\frac{1-x}{x}\right)^{p-1}\left(\frac{\rho_G}{\rho_L}\right)^{q-1}\left(\frac{\mu_L}{\mu_G}\right)^{r}. \quad [23]$$

The values of A, p, q and r for the different correlations are given in table 1.

CONCLUSIONS

This note shows that a number of apparently dissimilar correlations are, in fact, very similar. It is tentatively suggested that [21] is used as the basis for a new void fraction correlation. This would require determining the constants A, p, q and r by fitting [21] to experimental data. However, two of these constants may be removed on physical grounds. It is reasonable to suppose that, for the extreme case when $\rho_G = \rho_L$ and $\mu_G = \mu_L$, we have $\alpha = x$. For this to be so, both A and p must be unity.

One possible drawback to the use of [21] with $p = 1$ can be seen if we examine [23]. It is evident that $p = 1$ implies that the slip ratio is independent of quality. This would put a limitation on the flexibility of the correlation which is not necessarily borne out by the data.

A further limitation of this form of correlation is that we have not allowed for any effect of flow rate on the void fraction. Again this may not reflect the data.

As with most correlations of this type, the proposed correlation method neglects the possible effects of the following factors:

(1) Flow regime;

(2) Inclination of the channel;

(3) Rate of change of quality with length due to evaporation or condensation.

It would, therefore, be advisable to investigate whether these effects are important and, if so, whether they could be accounted for by having different values of A, p, q and r for different conditions.

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